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Axion Cosmology with a Stronger QCD in the Early Universe

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Abstract

We examine in the context of supersymmetric models whether the usual cosmological upper bound on the axion decay constant can be relaxed by assuming a period of stronger QCD in the early universe. By evaluating the axion potential in the early universe and also taking into account the dilaton potential energy, it is argued that a stronger QCD is not useful for raising up the bound.

I. INTRODUCTION

One of the most attractive solutions to the strong CP problem is to introduce a spontaneously broken Peccei-Quinn symmetry [1]. This solution originally predicted a pseudo-Goldstone boson, the so-called Peccei-Quinn-Weinberg-Wilczek axion [2], which is ruled out phenomenologically. The subsequently devised invisible axion [3] is also constrained to a narrow band on the axion decay constant f_a. Some astrophysical arguments, e.g. those associated with the axion emission from helium burning red giants or the supernova SN1987A [4,5], imply that f_a is far above the weak scale. A natural possibility is then that f_a is around the grand unification scale or the Planck scale. As is well known, however, the coherent axion oscillation in the early universe gives rise to relic axions whose mass density Ω_a (in the unit of the critical energy density) at present is given by [6]

$$\Omega_a = (\delta a/f_a)^2 (f_a/4 \times 10^{12}\,\mathrm{GeV})^{1.18}, \tag{1}$$

where δa is the axion misalignment at the epoch of the QCD phase transition. Clearly it is most natural to assume that the axion misalignment is of order f_a. Requring the axion mass density not significantly exceed the critical density, this leads to the upper bound $f_a \leq 4 \times 10^{12}$ GeV which is far below the grand unification scale or the Planck scale.

One way to raise up the cosmological upper bound on f_a is to have an entropy production after the QCD phase transition [7]. However it has been argued that, due to constraints from the nucleosynthesis, one cannot reach above few times 10^{15} GeV by this late time entropy production [8]. Recently another interesting way to raise up the axion bound was suggested by Dvali [9]. The idea is to suppress the axion misalignment δa by assuming a period of stronger QCD in the early universe during which the QCD coupling constant takes a value larger than the present one. Clearly this would be possible only when the QCD coupling constant is determined by a dynamical degree of freedom in the theory, e.g. a dilaton field ϕ whose vacuum expectation value (VEV) gives $1/g^2 = \langle\phi\rangle$. If the dilaton takes a value smaller than the present one, the QCD at that time would be stronger, leading

to the corresponding scale Λ_{QCD} *far bigger* than the present value of order 0.2 GeV. Then there is a possibility that a large effective axion mass is induced in the early universe, driving the axion field toward the minimum of the effective potential. If the minimum of the early axion potential coincides with the minimum of the present axion potential, δa would be dynamically suppressed, allowing the upper bound of f_a larger than 4×10^{12} GeV. Note that Eq. (1) indicates that the upper bound of f_a can be around $10^{16}\sim 10^{18}$ GeV if $\delta a/f_a$ is relaxed down to $10^{-2}\sim 10^{-3}$ in the early universe.

In order for the axion misalignment δa to be relaxed by the above mechanism, one needs

$$(i)\quad m_a \geq H, \qquad (ii)\quad |\langle a/f_a\rangle - \theta_{\rm eff}| \leq 10^{-3}\sim 10^{-2}, \tag{2}$$

where m_a and $\langle a\rangle$ denote the axion mass and the vacuum expectation value (VEV) determined by the axion potential in the early universe, and $\theta_{\rm eff}$ is the low energy QCD vacuum angle *at the present*. Here the first condition is required for the onset of the axion field oscillation toward the minimum, and the second is for the location of this minimum to coincide with the present one up to a small misalignment angle of order $10^{-3}\sim 10^{-2}$. Note that the present axion VEV is given by the low energy vacuum angle $\theta_{\rm eff}$ which includes the contributions from a variety of CP-odd parameters in the underlying theory. For instance, if the underlying theory is the minimal supersymmetric standard model,

$$\theta_{\rm eff} = \theta_{QCD} + \arg(\det(\lambda_u\lambda_d)) + 3\arg(m_{1/2}) - 3\arg(\mu B), \tag{3}$$

where θ_{QCD} denotes the *bare* QCD vacuum angle, λ_u and λ_d are the Yukawa coupling matrices, and $m_{1/2}$, μ and B denote the gluino mass, the Higgs μ parameter, and the soft B parameter, respectively. We stress that the second condition in Eq. (2) for the early minimum to coincide with the present one should be satisfied *without* fine tuning of some field values which would be *chaotic* in the early universe. Such fine tuning is certainly nothing better than the simpler fine tuning of the chaotic axion field a/f_a to $\theta_{\rm eff}$ at the epoch of QCD phase transition.

In this paper, we wish to examine whether the axion misalgnment can be relaxed by a stronger QCD in the early universe in supersymmetric models. To start with, we assume

that the dilaton dynamics in the model allows a period in the early universe during which QCD becomes stronger. The reason for limiting the analysis to supersymmetric models is clear: the most natural playground of the dilaton field is supersymmetric model.

The organization of this paper is as follows. In Section 2, we provide a discussion of the axion potential in the early universe by distinguishing the two cases, one in which Λ_{QCD} is smaller than the size of soft supersymmetry breaking parameter $m_{\rm soft}$ and the other opposite case that the QCD becomes so strong that $\Lambda_{QCD} \gg m_{\rm soft}$. We discuss the second case in somewhat detail since the axion potential in this case has not been considered in the previous literature. As we will see, the axion potential in the second case dramatically differs from the well known axion potential in the first case. Based on this analysis, in Section 3 we examine whether the two conditions given in Eq. (2) can be simultaneously met. Taking into account the dilaton potential energy, we will argue that it is very *unlikely* to have a cosmological scenario in which both of these two conditions are satisfied. We thus conclude in Section 4 that a stronger QCD in the early universe is *not* useful for raising the cosmological upper bound of the axion decay constant f_a.

II. AXION POTENTIAL IN THE PERIOD OF STRONGER QCD

In this section, we present a somewhat detailed discussion of the axion potential in the early universe. In the early universe, field variables would take chaotic values if their effective masses are smaller than the expansion rate H. In order to satisfy Eq. (2) without fine tuning, one needs to keep the axion VEV *not* shifted by such chaotic fields. If not, the axion VEV in the early universe would become chaotic and thus not coincide with the present axion VEV in general. We will thus assume that field variables which apparently affect the axion VEV are *not* chaotic, *but* quickly settle into the minimum of their effective potential in the early universe. Of course, one still needs to confirm that these fields settled at the minimum leads to an early axion VEV almost the same as the present one as given in Eq. (2).

To compute the axion potential in the early universe, it is convenient to distinguish the following two cases:

$$(i) \quad \Lambda_{QCD} < m_{\rm soft}, \qquad (ii) \quad \Lambda_{QCD} \gg m_{\rm soft}.$$

Let us first consider the case that $\Lambda_{QCD} < m_{\rm soft}$. In this case, if there is no light quark whose current mass is less than Λ_{QCD}, the axion potential is given by

$$V_a \simeq \Lambda_{QCD}^4 \cos(a/f_a - \theta_{\rm in}), \tag{4}$$

where $\theta_{\rm in}$ denotes the QCD vacuum angle *in the early universe*. The presence of light quarks suppresses the axion potential as

$$V_a \simeq m_q \Lambda_{QCD}^3 \cos(a/f_a - \theta_{\rm in}), \tag{5}$$

where m_q $(< \Lambda_{QCD})$ is the current mass of the *lightest* quark.

Of course, it depends upon the Higgs VEVs in the early universe whether there exists a light quark with $m_q < \Lambda_{QCD}$. In fact, the Higgs VEVs in the early universe can dramatically differ from the present ones. To be explicit, let us consider the minimal supersymmetric standard model. In this model, the Higgs potential for the neutral components takes the form

$$m_1^2|H_u|^2 + m_2^2|H_d|^2 + (B\mu H_u H_d + {\rm h.c.}) + \frac{1}{8}(g_2^2 + g_1^2)(|H_u|^2 - |H_d|^2)^2, \tag{6}$$

where H_u and H_d denote the two Higgs doublets in the model. In generic supersymmetric models, soft parameters are determined by the VEVs of fields triggering spontaneous SUSY breaking. As a result, soft parameters in the early universe may take values which differ from the present ones. In particular, both the sign and magnitude of m_1^2 and m_2^2 in the early unverse can differ from the present ones due to an additional SUSY breaking by nonzero energy density. This change of m_1^2 and m_2^2 does not alter $\arg(H_u H_d)$ and thus *not* lead to a change of the early axion VEV. However it can dramatically change $|H_u|$ and $|H_d|$. For instance, if both m_1^2 and m_2^2 are positive and large enough, the Higgs VEVs do vanish. Then

all quarks become massless and the axion potential vanishes up to a tiny contribution from small instantons. In the opposite case that both m_1^2 and m_2^2 are negative, the Higgs VEVs slide down toward the flat direction $|H_u| = |H_d| \gg m_{\rm soft}$. In this case, we would not have any light quark and the axion potential is given by (4). Finally for m_1^2 and m_2^2 leading to the Higgs VEVs similar to the present ones, the axion potential is given by (5) with the up quark mass $m_q \simeq 10$ MeV.

In the above, we have ignored a possible finite temperature effect on the axion potential. At high temperature $T > \Lambda_{QCD}$, the axion potentials Eq. (4) and Eq.(5) are *not* valid anymore. However, compared to the low temperature result, the high temperature axion potential is suppressed by more powers of the light quark masses and also by $\exp(-8\pi^2/g^2(T))$ [10]. Since what we wish to obtain is a bigger axion potential for a given value of Λ_{QCD}, we will not consider such a high temperature case anymore.

In the above discussion, the Higgs fields were assumed to stay at the minimum of the potential. It has been argued that in generic supergravity models a nonzero energy density $\rho_E \sim H^2 M_{\rm Pl}^2$ in the early universe affects soft parameters as $\delta m_{\rm soft} = cH$ where H is the Hubble expansion rate and c is a dimensionless constant [11]. The constant c may be small due to small couplings or to loop suppression. If c is small enough, it is possible that $m_{\rm soft} \ll H$. Such a large H may result in chaotic Higgs values in the early universe. However, in supersymmetric models with two Higgs doublets H_u and H_d, chaotic Higgs fields lead to a chaotic axion VEV which differs from the present axion VEV in general. Furthermore, for $H \geq m_{\rm soft}$, we have $m_a \ll H$ (Note that we have been discussing the case $\Lambda_{QCD} < m_{\rm soft}$.) and thus the first condition of Eq. (2) can *not* be satisfied also. We thus assume that $H \leq m_{\rm soft}$ and thus the Higgs fields quickly settle into their VEVs.

Let us now consider the opposite limit that the QCD becomes so strong that $\Lambda_{QCD} \gg m_{\rm soft}$. Then the axion potential is induced by nonperturbative supersymmetric QCD effects. Recent progress in understanding the dynamics of supersymmetric gauge theories [13] allows us to estimate the axion potential even in this case.

As is well known, the axion potential is severely affected by gauge invariant condensates

which break chiral symmetries in the theory. These condensates can be used to tie together fermion zero modes of the instanton amplitudes generating the axion potential. Phase degrees of freedom of such condensates then mix with the axion field as the η' mixes with the axion in normal QCD [4]. We thus study first what kind of chiral symmetry breaking condensates are formed in the supersymmetric QCD with the superpotential

$$W = \lambda_u H_u Q u^c + \lambda_d H_d Q d^c, \tag{7}$$

where Q, u^c, and d^c denote $SU(2)_L$ doublet quarks and singlet antiquarks, respectively. Here, the generation indices and gauge group indices are omitted for simplicity.

It has been argued that, for a vanishing superpotential, the quantum moduli space of degenerate vacua for supersymmetric $SU(3)_c$ gauge theory with $N_f = 6$ quark flavors is the same as the classical one [13]. Such a vacuum degeneracy is lifted in fact by the Yukawa terms in the superpotential and also by soft breaking terms [14]. As was observed recently, the infrared behavior of gauge invariant operators in supersymmetric $SU(N_c)$ gauge theory with N_f quark flavors can be studied by the dual $SU(N_f - N_c)$ theory with N_f dual quarks [13]. The superpotential of the dual model contains

$$W_D = \Lambda_{QCD}(\lambda_u H_u T_u + \lambda_d H_d T_d) + T_u Q_D u^c_D + T_d Q_D d^c_D, \tag{8}$$

where $T_u = Qu^c/\Lambda_{QCD}$ and $T_d = Qd^c/\Lambda_{QCD}$ denote the meson superfields with dimension one at the ultraviolet, while Q_D, u^c_D, and d^c_D are the dual quark and anti-quark superfields. Once the supersymmetry of the original model is broken by soft parameters much less than Λ_{QCD}, the dual model will contain soft terms [14]

$$\mathcal{L}^{(D)}_{\rm soft} = AW_D + \sum m_I^2 |\phi_I|^2, \tag{9}$$

where ϕ_I denotes generic scalar fields in the dual model and the soft parameters A and m_I are again much less than Λ_{QCD}.

A nice feature of the dual theory is that it becomes weaker as the original QCD becomes stronger, allowing a classical approximation to be valid. The classical effective potential

of the dual model can be readily computed from the superpotential (8) and the soft terms (9). It is then easy to see that, for Λ_{QCD} large enough compared to $m_{\rm soft}$, the Higgs fields $H_{u,d}$ and the linear combinations $\lambda_u T_u$ and $\lambda_d T_d$ of mesons have *positive* mass squared of order Λ^2_{QCD}, and thus have *vanishing* VEV's. For the original squarks having *positive* soft mass squared, it is expected that both the mesons and the dual squarks also have positive soft mass squared. Note that the mesons are the bound states of the original squarks and anti-squarks, while the dual squarks can be obtained by dissociating the scalar baryons (containing N_c original squarks) into $(N_f - N_c)$ pieces. With positive soft mass squared, the VEVs of the dual squarks and the entire mesons *vanish* also.

The above results on the VEVs can be summarized as

$$\begin{aligned}
\langle \tilde{q}\tilde{q}^c \rangle &\propto \langle T \rangle = 0, \\
\langle q q^c \rangle &\propto \langle F_T \rangle = 0, \\
\langle \lambda\lambda \rangle &\propto \langle T \rangle^{N_f/(N_f-N_c)} = 0,
\end{aligned} \tag{10}$$

where T denotes the scalar components of the meson superfields $T_{u,d}$ with the auxiliary components F_T. Thus all of the squark condensates $\langle \tilde{q}\tilde{q}^c \rangle$, the quark condensates $\langle q q^c \rangle$, and the gluino condensate $\langle \lambda\lambda \rangle$ do vanish. Note that the vanishing of the gluino condensate is essentially due to the vanishing Higgs VEV leading to the massless quarks. Although the above chiral invariant vacuum configurations were derived based on the classical potential at zero temperature, it is rather easy to see neither quantum corrections (due to the weak dual gauge interactions) nor finite temperature effects does shift these chiral invariant configurations.

So far, we have argued that the mesons and the dual squarks have vanishing VEVs for $\Lambda_{QCD} \gg m_{\rm soft}$. Again if the expansion rate H is large enough so that $H \gg m_{\rm soft}$, the mesons and dual squarks would have chaotic values although their effective potential is minimal at zero values. Note that the dual squarks and the mesons (except for $\lambda_u T_u$ and $\lambda_d T_d$) have masses of order $m_{\rm soft}$. Chaotic values of the mesons and the dual squarks correspond to

chaotic condensates of the quarks, squarks, and gluino. Clearly the axion VEV determined by such chaotic condensates will be *chaotic* also, and thus does *not* coincide in general with the present axion VEV. Thus to avoid a chaotic axion VEV, we assume that $m_{\rm soft} \geq H$ and the mesons and the dual squarks quickly settle into the minimum of their effective potential.

In the case that there is *no* condensate, the axion potential is given by the QCD instanton amplitudes whose fermion zero modes are tied together by the interactions which break the chiral symmetries explicitly [15]. To proceed, let us consider the minimal supersymmetric standard model having the superpotential

$$W = \lambda_u H_u Q u^c + \lambda_d H_d Q d^c + \mu H_u H_d, \tag{11}$$

and the soft terms

$$\mathcal{L}_{\rm soft} = \frac{1}{2} m_{1/2} \lambda\lambda + A(\lambda_u H_u \tilde{Q} \tilde{u}^c + \lambda_d H_d \tilde{Q} \tilde{d}^c) + B\mu H_u H_d + \frac{1}{2} \sum m_i^2 |\phi_i|^2 + \text{h.c.}, \tag{12}$$

where ϕ_i denotes generic scalar fields in the model. In order to compute the instanton-induced axion potential, we first note that the model is invariant under

$$G_{MSSM} = SU(3)_Q \times SU(3)_{u^c} \times SU(3)_{d^c} \times U(1)_A \times U(1)_X \times U(1)_R, \tag{13}$$

where the fields and parameters transform as $Q = (3, 1, 1)$, $u^c = (1, \bar{3}, 1)$, $d^c = (1, 1, \bar{3})$, $\lambda_u = (\bar{3}, 3, 1)$, and $\lambda_d = (\bar{3}, 1, 3)$ under $SU(3)_Q \times SU(3)_{u^c} \times SU(3)_{d^c}$, and the quantum numbers of $U(1)_A \times U(1)_X \times U(1)_R$ are given in Table 1. As is well known, the soft supersymmetry breaking can be described within superspace formalism by introducing spurion superfields:

$$\eta = \{1 + m_i^2 \theta^2 \bar{\theta}^2\},$$

$$Y = (1 + 16\pi^2 m_{1/2} \theta^2)\tau,$$

$$Z = \{Z_{u,d} = (1 + A\theta^2)\lambda_{u,d}, \quad Z_\mu = (1 + B\theta^2)\mu\},$$

where the auxiliary components of the spurion superfields represent soft breaking, while the scalar components denote supersymmetric couplings: the complex gauge coupling $\tau = \frac{8\pi^2}{g^2} + i\theta_{QCD}$, the Yukawa couplings $\lambda_{u,d}$, and the μ parameter. Note that the factor $16\pi^2$ in

Y is introduced for a proper normalization of the gluino mass $m_{1/2}$. Obviously η's are real superfields while Y and Z's are chiral.

After integrating out the gauge and matter superfields, the axion effective Lagrangian can be read off from the effective lagrangian of the spurions by identifying the dimensionless axion superfield

$$A = (s + ia + \tilde{a}\theta + F_A\theta^2)/f_a$$

as the fluctuation of Y, i.e. by the identification

$$Y \to Y + A. \tag{14}$$

The effective Lagrangian of spurions can be written as

$$\int d^2\theta d^2\bar{\theta}\, K_{\rm eff}(Y, Y^*, Z, Z^*, \eta) + \int d^2\theta\, W_{\rm eff}(Y, Z) + \text{h.c.}, \tag{15}$$

where the effective Kahler potential $K_{\rm eff}$ is a real function of spurion superfields (and also of their supercovariant derivatives) while the effective superpotential $W_{\rm eff}$ is a holomorphic function of chiral spurions.

Since the matter and gauge fields are integrated over a unique ground state preserving chiral symmetries, the above effective Lagrangian does *not* have any branch cut associated with the multiplicity of ground states. Note that in the case with nonvanishing gaugino condensate $\langle\lambda\lambda\rangle \sim e^{-i\theta_{QCD}/N}$, the corresponding effective Lagrangian contains terms depending upon $e^{-Y/N} \sim e^{-i\theta_{QCD}/N}$ and thus has N branches associated with the N degenerate vacua which are related to each other by the 2π shift of θ_{QCD} [16]. In our case with a unique chiral invariant vaccum, the effective Lagrangian is a single valued function of $e^{-i\theta_{QCD}}$, and thus is manifestly periodic under the 2π shift of θ_{QCD}.

Instantons would induce a term in $W_{\rm eff}$ as

$$e^{-nY}\omega(Z), \tag{16}$$

where n is a positive integer corresponding to the instanton winding number. It is, however, easy to see that the selection rules of G_{MSSM} does not allow any holomorphic ω which is

finite at $\mu \to 0$. This implies that the axion potential does *not* appear through $W_{\rm eff}$, *but* through $K_{\rm eff}$. Again the selection rules of G_{MSSM} implies that at the leading order the instanton-induced Kahler potential takes the form (for $n = 1$):

$$K_{\rm eff} \propto e^{-Y} {\rm det}(Z_u Z_d) Z_\mu^{*3} F(\eta) + {\rm h.c.}, \tag{17}$$

where F is an arbitrary function of the real superfields η.

To obtain the axion potential from the superspace integration of $K_{\rm eff}$, we need at least either a single insertion of D_η or the simultaneous insertions of $F_{Y,Z}$ and $F^*_{Y,Z}$ where D and F denote the auxiliary components of spurions. In other words, supersymmetry (together with the selection rules of G_{MSSM}) implies that the axion potential is suppressed at least by the two powers of $m_{\rm soft} = \{m_i, m_{1/2}, A, B\}$. Note that here we consider instantons whose scale, i.e. the inverse of the instanton size ρ, is in the range between $m_{\rm soft}$ (or μ) and the messenger scale M_m above which soft breaking is *not* operative anymore. In the popular hidden sector models in which supersymmetry breaking is transmitted by supergravity interactions [17], the messenger scale $M_m = M_{\rm Pl}$, while it can be much lower in visible sector models [18]. Obviously instantons with $\rho < M_m^{-1}$ (if exist) do not contribute to the axion potential due to the restored SUSY. As we will see, instantons which give dominant contributions have size $\rho \sim \Lambda_{QCD}^{-1}$ and thus belong to the above category for $\mu \sim m_{\rm soft} \ll \Lambda_{QCD}$. At any rate, from the Kähler potential of Eq. (17), we readily find the SUSY suppression factor

$$[m_{\rm soft}]^2 = \{m_i^2, AB^*, 16\pi^2 m_{1/2} B^*\}, \tag{18}$$

where the factor $16\pi^2$ in front of $m_{1/2}$ indicates that instanton graphs using $m_{1/2}$ to tie together gluino zero modes contain one less loop compared to those using other soft parameters.

With the above observation, one can write the axion potential in the case $\Lambda_{QCD} \gg m_{\rm soft}$ as

$$V_a = e^{ia/f_a} \mu^{*3} {\rm det}(\lambda_u \lambda_d)\, \Omega + {\rm h.c.} \tag{19}$$

where Ω is suppressed by $[m_{\rm soft}]^2$ and also by some powers of the loop factor $1/16\pi^2$. One may estimate Ω using an explicit instanton graph. For instance, the dimensional analysis rule of Ref. [15] applied for the graph of Fig. 1 yields a rough estimate:

$$\Omega \simeq (\frac{1}{16\pi^2})^6 \int d\rho\, f(\rho, M)\, [m_{\rm soft}]^2 \exp[-8\pi^2/g^2(\rho)], \tag{20}$$

where $[m_{\rm soft}]^2 = 16\pi^2 m_{1/2} B^*$ and f is a *dimensionless* function of the instanton size ρ and the masses $\{M\}$ of quantum fluctuations in the graph. If $\rho M \leq 1$ for all fluctuations, f would be of order unity, while it is suppressed by some powers of $1/\rho M$ when $\rho M \gg 1$ for some of fluctuations. We already noted that the Higgs masses are of order Λ_{QCD}. For $\rho \ll \Lambda_{QCD}^{-1}$, the negative beta function of supersymmetric $SU(3)$ theory with $N_f = 6$ is not negligible, implying $\exp[-8\pi^2/g^2(\rho)]$ leads to a significant suppression. For larger instantons with $\rho \gg \Lambda_{QCD}^{-1}$, although $\exp[-8\pi^2/g^2(\rho)]$ is roughly a constant since the beta function is small enough to approach to the fixed point [13], f is suppressed by the large Higgs masses. The above arguments imply that the dominant contribution is from instantons with size $\rho \sim \Lambda_{QCD}^{-1}$. We thus have

$$V_a \simeq e^{ia/f_a} (\frac{1}{16\pi^2})^6 \mu^{*3} {\rm det}(\lambda_u \lambda_d) [m_{\rm soft}]^2 \Lambda_{QCD}^{-1} + {\rm h.c.} \tag{21}$$

where $[m_{\rm soft}]^2$ is given in Eq. (18).

In the above, we have computed the axion potential for the MSSM with $\Lambda_{QCD} \gg m_{\rm soft}$. In fact, the MSSM can be considered as a rather special case since it contains a supersymmetric dimensionful parameter μ. To see what happens in models without such a parameter, let us consider the next minimal supersymmetric standard model (NSSM) including an additional gauge singlet S with the superpotential

$$W = \lambda_u H_u Q u^c + \lambda_d H_d Q d^c + \lambda_1 S H_u H_d + \lambda_2 S^3. \tag{22}$$

As the MSSM, one can use the supersymmetry and also the selection rules of

$$G_{NSSM} = SU(3)_Q \times SU(3)_{u^c} \times SU(3)_{d^c} \times U(1)_A \times U(1)_X \times U(1)_{X'} \times U(1)_R,$$

to constrain the effective Kahler potential $K_{\rm eff}$ leading to the axion potential. (For the quantum numbers of G_{NSSM}, see Table 2.) We then find $K_{\rm eff}$ is proportional to

$$e^{-Y}{\rm det}(Z_u Z_d)Z_1^{*3}D^2 Z_2 F(\eta), \tag{23}$$

where the new spurion superfields Z_1 and Z_2 are defined as

$$Z_1=(1+A\theta^2)\lambda_1, \quad Z_2=(1+A\theta^2)\lambda_2,$$

and $D^2 = D_\alpha D^\alpha$ denotes the supercovariant derivative. Although D^2 is applied to Z_2 in the above example, it can be applied to other spurions also. For a rough estimate, one may consider Fig. 2. Again applying the dimensional analysis rule given in Ref. [15], we find

$$V_a \simeq e^{ia/f_a}(\frac{1}{16\pi^2})^8{\rm det}(\lambda_u\lambda_d)\lambda_1^{*3}\lambda_2 A\,[m_{\rm soft}]^2\Lambda_{QCD}+{\rm h.c.}, \tag{24}$$

where again $[m_{\rm soft}]^2$ is given in Eq. (18).

III. RELAXATION OF THE AXION MISALIGNMENT

In the previous section, we have estimated the axion potential by distinguishing the two cases: (i) $\Lambda_{QCD} < m_{\rm soft}$ and (ii) $\Lambda_{QCD} \gg m_{\rm soft}$. Based on this, in this section we examine whether the axion misalignment can be relaxed to a small value by a stronger QCD in the early universe. To proceed, let us note that in generic supergravity models some low energy parameters other than Λ_{QCD} are determined also by the VEVs of some fields. In the early universe, such parameters may take values which differ from the present ones. However, in order for the early axion VEV to coincide with the present VEV, one should require that complex parameters which affect the axion VEV have almost the same values as the present ones. For the MSSM,

$$(\theta_{\rm eff})_{MSSM}=\theta_{QCD}+{\rm arg}({\rm det}(\lambda_u\lambda_d))+3{\rm arg}(m_{1/2})-3{\rm arg}(\mu B), \tag{25}$$

while for the NSSM we have

$$(\theta_{\rm eff})_{NSSM} = \theta_{QCD} + \arg(\det(\lambda_u\lambda_d)) + 3\arg(m_{1/2}) - 2\arg(A) - \arg(\lambda_1^3\lambda_2^*). \tag{26}$$

This leads us to assume that in the early universe all Yukawa couplings, μ, and complex soft parameters (A, B, $m_{1/2}$) have the same values as the present ones in order that the early axion VEV can coincide with the present VEV.

Clearly the energy density in the early universe contains the dilaton potential energy $V(\phi)$. At present with $\phi = \phi_0$, we have $V(\phi_0) = 0$. However, in the early universe with $\phi \neq \phi_0$ if the dilaton potential is *not* flat, which is believed to be the case, $V(\phi)$ would have a *nonzero* positive value. This means that raising up Λ_{QCD} to raise up the axion mass m_a raises up also the energy density and thus the expansion rate H. In supersymmetric models, it is convenient to parameterize this dilaton potential energy in the early universe as follows:

$$V(\phi) = C m_{\rm soft}^2 M_m^2, \tag{27}$$

where C is a dimensionless coefficient and M_m is the messenger scale of SUSY breaking above which the soft SUSY breaking is not operative anymore and thus a precise SUSY cancellation takes place. To be more concrete, here we specify $m_{\rm soft}$ as the size of soft SUSY breaking for supermultiplets in the supersymmetric standard model sector.

Since it is relevant for our later discussion, let us estimate the size of C. First of all, if the dilaton potential is generated directly by the SUSY breaking dynamics, $V(\phi)$ is expected to be of order $|F|^2$ where F denotes the auxiliary components of generic fields in SUSY breaking sector. For hidden sector models in which this SUSY breaking is transmitted by supergravity interactions, we have $M_m \simeq M_{\rm Pl}$ and $m_{\rm soft} \simeq |F|/M_{\rm Pl}$ [17], and thus C is of order unity. For visible sector models in which the SUSY breaking is transmitted by gauge interactions [18], the messenger scale corresponds to the scale of dynamical SUSY breaking and thus $M_m \simeq |F|^{1/2}$, while the soft breaking in the supersymmteric standard model sector is radiatively generated as $m_{\rm soft} \simeq (\frac{\alpha}{4\pi})^n |F|^{1/2}$ where n is a model-dependent positive integer. This implies that C is of order $(4\pi/\alpha)^{2n}$ for visible sector models when the dilaton potential energy directly induced by the SUSY breaking dynamics is parameterized as Eq. (27). In

summary, if the dilaton potential is generated directly by the SUSY breaking dynamics, C would be of order unity or bigger by $(4\pi/\alpha)^{2n}$.

Even in the case that the dilaton potential is *not* induced directly by the SUSY breaking dynamics, once SUSY is broken, it is generated in general by higher order loop effects. For instance, loops of colored particles would induce a dilaton-dependent, i.e. QCD coupling-dependent, contribution to the vacuum energy density. Such loop effects are quadratic in both the mass splitting in supermultiplets and the messenger scale M_m which corresponds to the cutoff scale, and thus can be written as Eq. (27). (Throughout this paper, the dilaton is minimally defined as a field whose VEV determines the QCD coupling. Of course this dilaton can affect other gauge couplings. Our whole discussion will be valid also for such general case.) In the hidden sector models, the mass splitting is of order $m_{\rm soft}$ which is independent of the QCD coupling at the leading order. As a result, a QCD coupling-dependence would not appear at one loop order, but does appear at two loops [19]. This implies that, due to radiative effects, C *cannot* be significantly smaller than $(\frac{1}{16\pi^2})^2$ in the hidden sector models. In visible sector models, things are a bit more complicated. Typical visible sector models include a so-called messenger sector which contains a vector-like quark and lepton superfields [18]. By the aid of messegner $U(1)$ gauge interaction, the SUSY breaking dynamics gives rise to a mass splitting δM in the messenger sector. Subsequent radiative effects of the standard model gauge interactions then lead to the mass splitting $m_{\rm soft} \simeq \frac{\alpha}{4\pi}\delta M$ in the supersymetric standard model sector. Since δM is independent of the QCD coupling at leading order, loops of the messenger sector particles will induce a QCD coupling-dependent vacuum energy density again at two loop order. The corresponding dilaton potential energy is of order $(\frac{1}{16\pi^2})^2(\delta M)^2 M_m^2$ which is of order $m_{\rm soft}^2 M_m^2$. Thus in visible sector models, again due to radiative effects, C *cannot* be significantly smaller than order unity.

With the dilaton potential energy given in Eq. (27) in the early universe, the Hubble expansion rate is given by

$$H \simeq C^{1/2} m_{\rm soft} M_m M_{\rm Pl}^{-1}, \tag{28}$$

where $M_{\rm Pl} = 2.44 \times 10^{18}$ GeV. Let us now examine whether the two conditions of Eq. (2) for the relaxation of the axion misalignment can be satisfied. We consider first the case that $\Lambda_{QCD} < m_{\rm soft}$ without any light quark. One can easily confirm that, as long as the complex parameters contributing to $\theta_{\rm eff}$ are unchanged, the axion VEV determined by the early axion potential Eq. (4), i.e. $\theta_{\rm in}$, coincides with $\theta_{\rm eff}$. In the previous section, we have noted that all quark masses would become larger than Λ_{QCD} if both of the Higgs soft masses m_1^2 and m_2^2 of Eq. (6) receives a significantly large *negative* contribution from the energy density in the early universe. This would make the Higgs VEVs slide down toward the flat direction $|H_u| = |H_d| \gg m_{\rm soft}$. (Note that a large positive contribution leads to vanishing Higgs VEVs.) However, as was noted in Ref. [12], such a negative contribution arises usually through supergravity interactions of the form $\frac{1}{M_{\rm Pl}^2}\int d^2\theta d^2\bar{\theta}\,\phi\phi^*\chi\chi^*$ where χ is an inflaton-like field yielding an energy density of order $H^2 M_{\rm Pl}^2$. For the above operators significantly alter the Higgs soft mass, one needs H to be comparable to $m_{\rm soft}$. Obviously then, we obtain $m_a \ll H$ since $\Lambda_{QCD} < m_{\rm soft} \ll f_a$ in this case.

Let us consider the next case that Λ_{QCD} is still less than $m_{\rm soft}$ but now there exists a light quark. The axion potential in this case is given by Eq. (5). Using this axion potential and Eq. (28), we then find

$$\frac{m_a}{H} \leq 5\, C^{-1/2} \left(\frac{4\times 10^{12}}{f_a}\right)\left(\frac{10^5}{M_m}\right)\left(\frac{m_q}{10^{-2}}\right)^{1/2}\left(\frac{\Lambda_{QCD}}{m_{\rm soft}}\right)^{3/2}\left(\frac{m_{\rm soft}}{10^2}\right)^{1/2}, \tag{29}$$

where all numbers in the brackets denote the energy scales in GeV unit. The messenger scale M_m can be as low as 10^5 GeV in the visible sector models for SUSY breaking. However, in other type of models including the popular hidden sector models, M_m is typically much bigger than 10^5 GeV. As was mentioned, complex parameters contributing to $\theta_{\rm eff}$ in the early universe are required to have the values which are the same as the present one in order for the early axion VEV ($\theta_{\rm in}$) to coincide with the present VEV ($\theta_{\rm eff}$). Real soft scalar masses m_i^2 escape from this requirement. They might be significantly bigger than the present values of order 10^2 GeV due to contributions from the radiation or other forms of energy density.

Note that if the early universe does not carry quantum numbers which are carried by some complex parameters, a nonzero energy density in such an early universe would enhance m_i^2 without affecting those complex parameters. However, if the enhanced soft mass-squared of Higgs fields are significantly bigger than the unaffected $B\mu \sim 10^2$ GeV, it makes the Higgs VEVs to vanish. This leads to $m_q = 0$ and thus dramatically suppresses the axion potential.

Taking into account the points discussed above, it is easy to note that, for a given value of f_a, the ratio m_a/H becomes maximal for Λ_{QCD} comparable to $m_{\rm soft}$ which is of order 10^2 GeV. Then m_a would be large enough to push the axion field toward the minimum if f_a is in the range: $f_a \leq (5C^{-1/2}) \times (10^5/M_m) \times (4 \times 10^{12})$ GeV. Clearly in the hidden sector models with $M_m = M_{\rm Pl}$, this range of f_a does not overlap with the interesting range $f_a \gg 4 \times 10^{12}$ GeV. In visible sector models, the messenger scale M_m corresponds to the scale of dynamical SUSY breaking and thus may be as low as 10^5 GeV. However as we have discussed, C is at least of order unity in visible sector models and thus the axion mass is *not* large enough to relax the axion misalignment for $f_a \gg 4 \times 10^{12}$ GeV.

So far, we have argued that raising Λ_{QCD} up to the order of $m_{\rm soft}$ or below (while keeping the complex parameters that contribute to $\theta_{\rm eff}$ unchanged) is not useful for relaxing the axion misalignment when $f_a \gg 4\times 10^{12}$ GeV. One might expect that raising Λ_{QCD} further up to far above $m_{\rm soft}$ can leads to an axion mass $m_a \geq H$. However, the axion potentials Eq. (21) and Eq. (24) computed in the previous section indicate that the axion mass is highly suppressed for $\Lambda_{QCD} \gg m_{\rm soft}$. This is mainly because the Higgs VEVs and the condensates of the quarks, squarks, and the gluinos do *not* become of order Λ_{QCD}, but they all *vanish* when $\Lambda_{QCD} \gg m_{\rm soft}$. As a result, instanton amplitudes for the axion potential are suppressed by the powers of small Yukawa couplings and also of the small mass parameters μ and $m_{\rm soft}$.

Before discussing the size of the axion mass, let us briefly discuss the axion VEV. As can be noticed, the axion VEV determined by the axion potentials Eq. (21) and Eq. (24) differ from the $\theta_{\rm eff}$ of Eq. (25) and Eq. (26) even when all complex parameters contributing $\theta_{\rm eff}$ in the early universe have the same values as the present one. In such a case, the misalignment angle $\delta\theta = \langle a/f_a \rangle - \theta_{\rm eff}$ for the MSSM is given by

$$(\delta\theta)_{MSSM} = N_1 \mathrm{arg}(m_{1/2}A^*) + N_2 \mathrm{arg}(m_{1/2}B^*), \tag{30}$$

where N_1 and N_2 are appropriate integers of order unity. Similarly for the NSSM, we have

$$(\delta\theta)_{NSSM} = N \mathrm{arg}(m_{1/2}A^*) \tag{31}$$

for an integer N of order unity. Although can be nonzero, the above misalgnment angle is severely constrained by the neutron electric dipole moment [20] as

$$\delta\theta \le 10^{-2} \sim 10^{-3}. \tag{32}$$

This would be small enough to raise the cosmological bound on f_a up to the grand unification scale or the Planck scale. However as we mentioned, the axion mass becomes too small to relax down the axion misalignment to a small value of order $\delta\theta$.

Again using the axion potentials Eq. (21) and Eq. (24), we find

$$(\frac{m_a}{H})_{MSSM} \simeq 10^{-11} C^{-1/2} \left(\frac{4\times 10^{12}}{f_a}\right)\left(\frac{10^5}{M_m}\right)\left(\frac{\mu}{10^2}\right)^{3/2}\left(\frac{10^2}{\Lambda_{QCD}}\right)^{1/2},$$

$$(\frac{m_a}{H})_{NSSM} \simeq 10^{-5} C^{-1/2} (\lambda_1^3\lambda_2)^{1/2} \left(\frac{4\times 10^{12}}{f_a}\right)\left(\frac{10^5}{M_m}\right)\left(\frac{A}{10^2}\right)^{1/2}\left(\frac{\Lambda_{QCD}}{M_{\rm Pl}}\right)^{1/2}, \tag{33}$$

where again the numbers in the brakets denote energy scales in GeV unit. The above results obviously indicate that $m_a \ll H$. Although only two models are explicitly considered for the case $\Lambda_{QCD} \gg m_{\rm soft}$, the huge suppression of the axion potential seems to be quite generic. In particular, adding more colored particles leads to a further suppression, and thus not helpful at all. We thus conclude that the axion misalignment *cannot* be relaxed down to a small value even when $\Lambda_{QCD} \gg m_{\rm soft}$. In summary, the analysis made in this section indicates that a stronger QCD in the early universe is not useful for relaxing the axion misalignment.

IV. CONCLUSION

In this paper, we have examined whether the axion misalignment can be relaxed down to a small value by a stronger QCD in the early universe. This would allow the axion

scale to be of order the grand unification scale or the Planck scale without any cosmological difficulty. We discussed in somewhat detail the axion potentials in the early universe, in particular for the case that $\Lambda_{QCD} \gg m_{\rm soft}$. Taking into account the dilaton potential energy associated with a stronger QCD, our analysis indicates that the two conditions $m_a \geq H$ and $\langle a/f_a \rangle = \theta_{\rm eff}$ (up to a small misalignment of order $10^{-3} \sim 10^{-2}$) *cannot* be satisfied simultaneously. We thus conclude that a stronger QCD in the early universe is not useful for raising the cosmological upper bound of the axion scale.

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TABLES

TABLE I. Quantum numbers of superfields and spurions in MSSM

	$U(1)_A$	$U(1)_X$	$U(1)_R$
Q	1	0	1
u^c, d^c	1	-1	1
H_u, H_d	0	1	0
e^{-Y}	12	-6	6
Z_u, Z_d	-2	0	0
Z_μ	0	-2	2
$d^2\theta$	0	0	-2

TABLE II. Quantum numbers of superfields and spurions in NSSM

	$U(1)_A$	$U(1)_X$	$U(1)_{X'}$	$U(1)_R$
Q	1	0	0	1
u^c, d^c	1	-1	0	1
H_u, H_d	0	1	0	0
S	0	0	1	0
e^{-Y}	12	-6	0	6
Z_u, H_d	-2	0	0	0
Z_1	0	-2	-1	2
Z_2	0	0	-3	2
$d^2\theta$	0	0	0	-2

FIGURES

FIG. 1. Instanton graph for the axion potential Eq. (21) of the MSSM. The solid lines with and without waves around the instanton denote the gluino and quark modes, respectively, while the dotted lines are the Higgs and squarks fluctuations. The dark blobs represent the insertions of complex couplings which are explicitly written in the graph. The vertices not marked with couplings are the QCD gauge couplings.

FIG. 2. Instanton graph for the axion potential Eq. (24) of the NSSM. Again the solid lines are for fermion modes , the dotted lines for boson fluctuations, and the dark blobs for the inserted complex couplings.